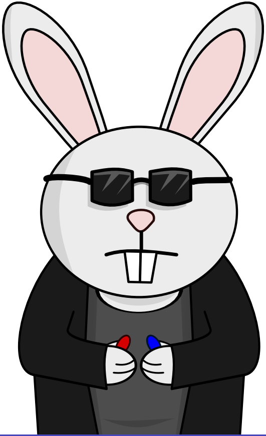
Maxwell's Demon and Landauer's Principle



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Entropy and Heat

- Mechanical ⇒ Heat
  - Energy is conserved (First Law)
  - entropy $\propto$ energy/heat
- Heat ⇒ Mechanical
  - Not all heat can be used (Second Law)
  - $\text{entropy}_{out} \geq \text{entropy}_{in}$
- What is entropy?
  - Amount of "invisible" information
  - Proportional to # bits required to describe atoms' position, velocity
  - More heat = more bits (10-20 per atom)

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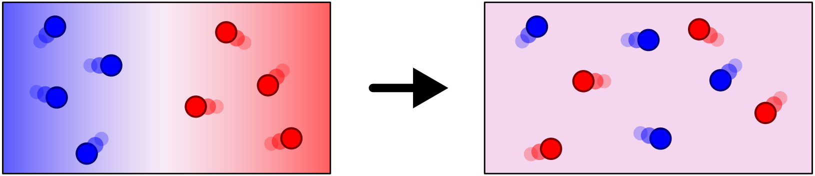
The Second Law

No process is possible whose sole result is the transfer of heat from a body of lower temperature to a body of higher temperature - Rudolf Clausius

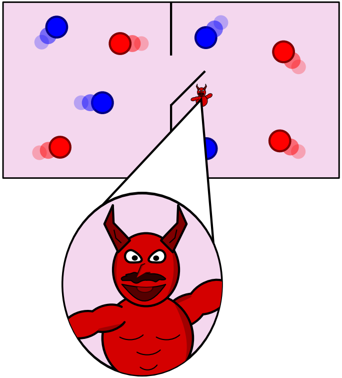


- Entropy increases (maximum at equilibrium)
- Information is preserved

Maxwell's Demon

... if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us. For we have seen that molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that such a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being, who can see the individual molecules, opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics.... - James Maxwell

Maxwell's Demon



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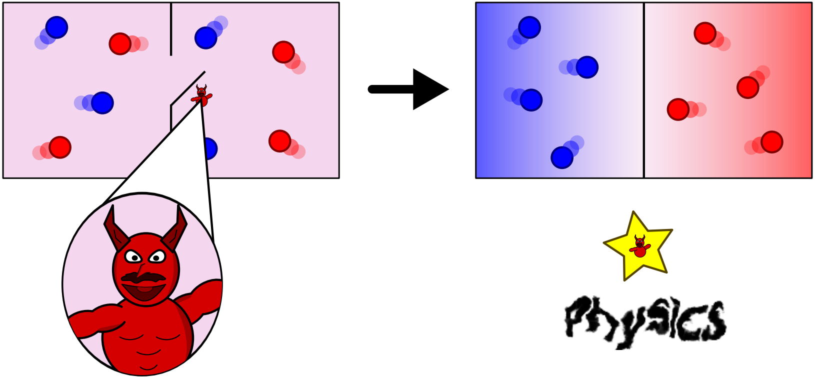
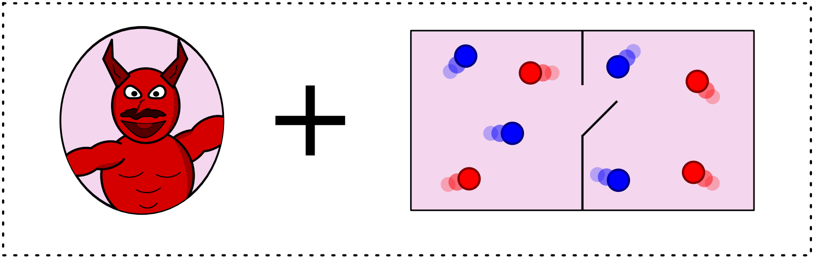


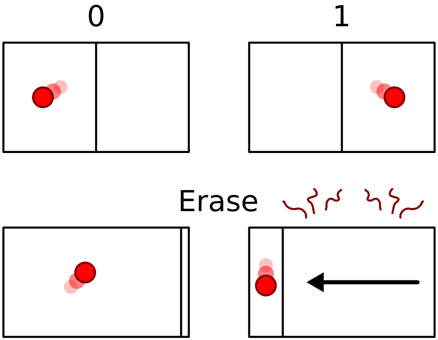
- Demon sorts gas molecules by velocity
- Physics is defeated!

Exorcising the Demon



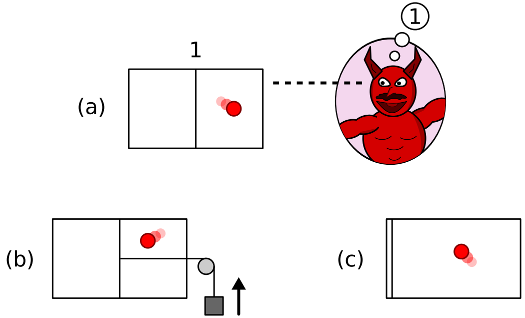
- Consider demon part of the system
- Demon must generate more entropy than can be eliminated

Toy System



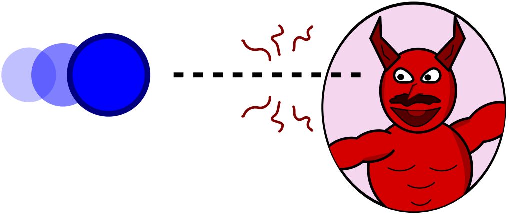
- System contains 1-bit of information
- Erasure produces $kT \ln 2$ heat (per bit)
  - Entropy decreases by $k \ln 2$

Toying with the Demon



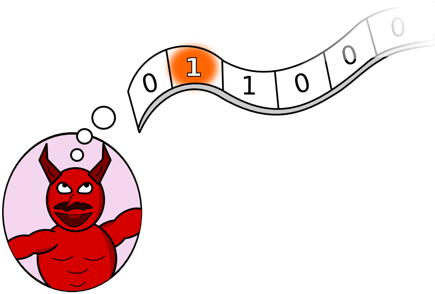
- Demon measures state (a), removes partition, inserts piston
- Gas expands, does $kT \ln 2$ work (b)
- System is reset (c)
- Profit!

Measurement Cost?



- Measuring requires energy expenditure (Leó Szilárd, 1929)
- Not if measurement is thermodynamically reversible (Rolf Landauer, 1960)

The Demon's Mind



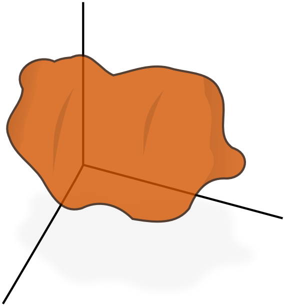
- Demon must erase bits eventually
- Erasure will balance out work
- Information is **physical**

Landauer's Principle

If the number of possible logical states decreases during computation, this must be compensated by an increase in the number of possible physical states per logical state.

- Computer's degrees of freedom
  - Information bearing (IBDF)
  - Non-information bearing (NIBDF)
- Logically irreversible operations
  - Decrease entropy of IBDF
  - Must increase entropy of NIBDF and environment (Second Law)
    - ⋆ May be reversible if applied to "random" data (?)

Phase Space Metaphor



- For a closed system
  - Phase space volume cannot decrease
  - Loss of states on one axis compensated by other axes

Generalized Entropy

- Difference (in bits) between thermodynamic entropy and observer's information
  - Thermodynamic system state now includes observer knowledge
  - Why include observer? Work can be extracted.
  - What does it mean to posses knowledge of a system?
- Heat $\Rightarrow$ Mechanical Energy
  - Nowhere to put excess information
  - $\approx 10^{25}$ bits for HE atoms in small balloon

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Principle Objections

1. Thermodynamics and logical reversibility have no connection
   - Substitute automatic mechanisms where intelligence is required
2. All data-processing operations require at least $kT \ln 2$ of energy
   - Practically true for macroscopic computers, but not theoretically
   - DNA to RNA transcription
3. Logically irreversible operations can be thermodynamically reversible
   - Merging of control flow is not reversible
4. Not sufficient to prove Second Law
   - Assuming the consequent (demon cannot violate Second Law)

- Pedagogical Purposes
  - Belief that information processing costs order kT energy (**wrong!**)
  - Acquisition, copying, and logical ops are free (if reversible)

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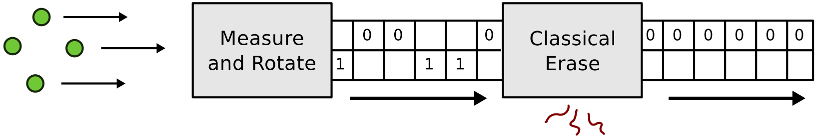
Quantum Erasure



1. Prepare particles in a quantum state with n outcomes having probabilities $\{p_i\}$
2. Measure each along $|e_i\rangle$, record outcomes
3. Using outcomes, rotate particles into pure state $|e_0\rangle$
4. Erase classical measurements
   - $kT \ln 2 H\{p_i\}$ heat generated
   - Can be optimized

$$H\{p_i\} = -\sum_{1}^{n} p_i \log(p_i)$$

Questions?